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FIRST DIGIT DISTRIBUTION OF HADRON FULL WIDTH*

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A phenomenological law, called Benford's law, states that the occurrence of the first digit,
i.e., $1, 2, ..., 9$, of numbers from many real world sources is not uniformly distributed, but
instead favors smaller ones according to a logarithmic distribution. We investigate, for
the first time, the first digit distribution of the full widths of mesons and baryons in the
well defined science domain of particle physics systematically, and find that they agree
excellently with the Benford distribution. We also discuss several general properties of
Benford's law, i.e., the law is scale-invariant, base-invariant, and power-invariant. This
means that the lifetimes of hadrons follow also Benford's law.

Keywords: Benford's law; first digit; hadron width; hadron life time.

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Many numbers in the real world follow a fascinating pattern of distribution
rather than a uniform distribution as might be expected. In 1881, Newcomb[1] noticed
that the preceding pages of the logarithmic table wear out faster, thus he hinted at
the idea that the first nonzero digit of many natural numbers favors small values.
Then in 1938, Benford[2] investigated a great number of data sets in various unrelated
fields, e.g., the arabic numbers on the front page of a newspaper, the wire and drill
gauges of the mechanic, the magnitude scales of astronomy, the street addresses
published in a magazine, the weights of molecules and atoms, the areas of lakes and
the lengths of rivers, and he found that they all agree with a logarithmic distribution
which now we refer to as Benford's law, or the first digit law,

$$P(k) = \log_{10}(1 + \frac{1}{k}), \ k = 1, 2, ..., 9 \tag{1}$$

where $P(k)$ is the probability of a number having the first nonzero digit k.

Surprisingly, there are plenty of unrelated data sets in the nature, e.g., phys-
ical constants[3], alpha-decay half-lives[4,5], survival distributions[6], the strengths of

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electric-dipolar lines in transition arrays of complex atomic spectra[7], and numbers in our everyday lives, e.g., the stock market indices[8], numbers in the World Wide Web[9], the file sizes in a personal computer[10], the winning bids for certain eBay auctions[11], all incredibly conform to the peculiar first digit law perfectly. Furthermore, Tolle et al.[12] provided empirical evidence that dynamical systems reveal the first digital behavior, and then Snyder et al.[13] developed corresponding computer simulations and confirmed the conclusion, and later it was discussed further by Berger et al.[14] in detail. Nevertheless, there also exist many data sets which do not obey the law, and unfortunately there is no priori criteria yet to judge which type a data set belongs to.

After the discovery for more than one century since 1881, a universally accepted explanation for the underlying reason of such a law is still lacking. However, many literatures have exploited a lot of applications of the logarithmic distribution in various fields, and the first digit law has been used in practice already. The main usage is to detect data and judge their reasonableness. It is applied in distinguishing and ascertaining fraud in taxing and accounting[15,16,17], fabrication in clinical trials[18], authenticity of the pollutant concentrations in ambient air[19], electoral cheats or voting anomalies[10], and falsified data in scientific experiments[20]. Moreover, the first digit law is largely applied in computer science for speeding up calculation[21], minimizing expected storage space[22], analyzing the behavior of floating-point arithmetic algorithms[23], and especially for various studies in the image domain[24,25].

Despite so many successful developments, there is no academic paper yet on the topic of Benford's law in the well defined science domain of particle physics, to the best of our knowledge. For the first time, we investigate the properties of particles by applying the first digit law, and find that in spite of the smallness of data capacity and the large span of the ranges, e.g., $\Gamma \sim 10^{-24}$ MeV for the neutron and $\Gamma \sim 10^3$ MeV for $f_0(600)$), the full widths of mesons and baryons pertain to the Benford distribution very well respectively. Hence the Benford's law applies to the full widths of hadrons with a rather good precision.

More specifically, we scrutinize the meson summary table and baryon summary table in Review of Particle Physics (2008) by Particle Data Group[26], and compare the first digit distribution of the full widths of mesons and baryons with Benford distribution respectively. The systematical statistical results are shown in Table 1 for mesons and Table 2 for baryons, respectively. The numbers in the bracket are the expected number,

$$N_{\text{Ben}} = N \log_{10}(1 + 1/k) \tag{2}$$

together with the root mean square error evaluated by the binomial distribution,

$$\Delta N = \sqrt{N P(k)(1 - P(k))}. \tag{3}$$

The detail of the classification from Case 1 to Case 3 will be discussed later. We can see that the results are very weakly case sensitive, and all are in good agreement

Table 1.   The first digit distribution of the full widths of mesons.

First Digit	Case 1 (88)		Case 2 (91)		Case 3 (96)	
1	24	(26.5±4.3)	25	(27.4±4.4)	25	(28.9±4.5)
2	22	(15.5±3.6)	22	(16.0±3.6)	22	(16.9±3.7)
3	11	(11.0±3.1)	11	(11.4±3.2)	12	(12.0±3.2)
4	9	(8.5±2.8)	11	(8.8±2.8)	12	(9.3±2.9)
5	5	(7.0±2.5)	5	(7.2±2.6)	7	(7.6±2.6)
6	2	(5.9±2.3)	2	(6.1±2.4)	2	(6.4±2.4)
7	5	(5.1±2.2)	5	(5.3±2.2)	5	(5.6±2.3)
8	6	(4.5±2.1)	6	(4.7±2.1)	6	(4.9±2.2)
9	4	(4.0±2.0)	4	(4.2±2.0)	5	(4.4±2.0)
Pearson χ^2	**6.62**		**6.82**		**6.32**	

Table 2.   The first digit distribution of the full widths of baryons.

First Digit	Case 1 (65)		Case 2 (72)		Case 3 (81)	
1	21	(19.6±3.7)	22	(21.7±3.9)	23	(24.4±4.1)
2	11	(11.4±3.1)	12	(12.7±3.2)	13	(14.3±3.4)
3	9	(8.1±2.7)	11	(9.0±2.8)	14	(10.1±3.0)
4	6	(6.3±2.4)	6	(7.0±2.5)	6	(7.8±2.7)
5	6	(5.1±2.2)	7	(5.7±2.3)	8	(6.4±2.4)
6	4	(4.4±2.0)	5	(4.8±2.1)	6	(5.4±2.2)
7	1	(3.8±1.9)	1	(4.2±2.0)	2	(4.7±2.1)
8	4	(3.3±1.8)	4	(3.7±1.9)	4	(4.1±2.0)
9	3	(3.0±1.7)	4	(3.3±1.8)	5	(3.7±1.9)
Pearson χ^2	**2.57**		**3.52**		**4.57**	

with Benford's law. The intuitive figures are illustrated in Fig. 1, with the left row for mesons, the right row for baryons for Case 1 to Case 3 from the top down.

It is worthy to mention that, when estimating the fitness to the theoretical probability distribution, we should use fitness estimating χ^2, namely Pearson χ^2,

$$\chi^2(n-1) = \sum_{i=1}^{n} \frac{(N_{\text{Obs}} - N_{\text{Ben}})^2}{N_{\text{Ben}}} \qquad (4)$$

where N_{Obs} is the observational number and N_{Ben} is the theoretical number from Benford's law, and here in our question $n = 9$. However, it is not appropriate to use parameter estimating χ^2 as used in Refs. 4, 5. In Eq. (4), the degree of freedom is $9 - 1 = 8$, and under the confidence level 95%, $\chi^2(8) = 15.507$, and under the confidence level 50%, $\chi^2(8) = 7.344$. The χ^2 we calculated is smaller than those in Refs. 4, 5, indicating clearly that the fitness is remarkably good in particle physics.

The classification from Case 1 to Case 3 is due to the incompleteness and uncertainty of experimental data, however, we do our best to treat data without bias. Since we only deal with the full widths of hadrons, we ignore the e^+e^- width Γ_{ee} and we do not distinguish the difference between the full width and the Breit-Wigner full width given in the baryon summary table. When the summary tables give only the lifetime τ instead of the full width Γ, we use $\Gamma \times \tau = \hbar$ to get the corresponding full width. And we drop the single-side data, e.g., $\Gamma < 1.9$ MeV for $\Lambda_c(2625)^+$, while we still keep the double-side data and pick the mean value of the boundaries,
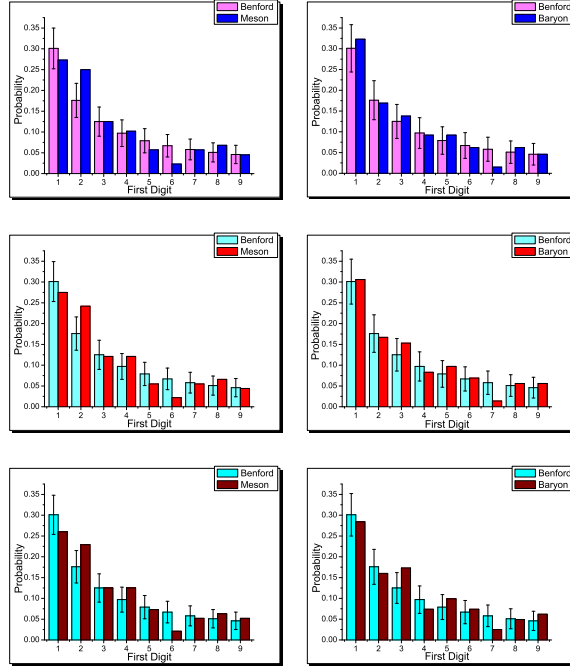
Fig. 1.   Comparisons of Benford's law and the distribution of the first digit of the full widths of mesons (left) and baryons (right).

for instance, given 200 to 500 MeV for $f_0(1370)$, we treat it as $\Gamma = 350$ MeV, and when given the most likely values in these two-boundary cases, we chose them for simplicity. But there still remain some puzzled data due to the isospin problem. For most hadrons of the same isospin I, there are several types of particles due to the different isospin projection I_3, and the summary table separates some while it does not distinguish between others definitely, however, the same isospin does not always promise the same lifetime hence the same full width. For the stringency of our approach, we use the following three schemes to deal with the data:

• In Case 1, we just drop out the puzzled data;
• In Case 2, we faithfully follow the classification published in the particle table and treat each item as a whole, and when there are several full widths appearing under one item, we pick the mean value and only count for once;
• And in Case 3, we stick to the appearance of the data, that is, when there is a datum, we count it for once.

Though the fitness is so impressive, we still feel short of data. Consequently, we add mesons and baryons up to get the distribution of hadrons. The results are

Table 3.   The first digit distribution of the full widths of hadrons.

First Digit	Case 1 (153)		Case 2 (163)		Case 3 (177)	
1	45	(46.1±5.7)	47	(49.1±5.9)	48	(53.3±6.1)
2	33	(26.9±4.7)	34	(28.7±4.9)	35	(31.2±5.1)
3	20	(19.1±4.1)	22	(20.4±4.2)	26	(22.1±4.4)
4	15	(14.8±3.7)	17	(15.8±3.8)	18	(17.2±3.9)
5	11	(12.1±3.3)	12	(12.9±3.4)	15	(14.0±3.6)
6	6	(10.2±3.1)	7	(10.9±3.2)	8	(11.8±3.3)
7	6	(8.9±2.9)	6	(9.5±3.0)	7	(10.3±3.1)
8	10	(7.8±2.7)	10	(8.3±2.8)	10	(9.1±2.9)
9	7	(7.0±2.6)	8	(7.5±2.7)	10	(8.1±2.8)
Pearson χ^2	**4.82**		**4.39**		**4.62**	

shown in Table 3 numerically, and in Fig. 2, Fig. 3 and Fig. 4 graphically. They all appear remarkably good.

Many attempts have been tried to explain the underlying reason for Benford's law. For theoretical reviews, see Ref. 27 and papers written by Hill[28,29,30]. We here present some discussions on the first digit law, focusing on its several general properties.

Firstly, a positive date set can be rewritten in the form of $\{10^{n_i+f_i}\}$, where n_i is an integer not affecting the first digit, and f_i ($0 \leq f_i < 1$) is the fractional part which contributes to the question. As suggested and derived by Newcomb[1], a uniform distribution of the fractional part f_i of the exponent in the interval $[0, 1)$ leads to the logarithmic law.

Secondly, as can be expected, multiplication of a constant upon the data set does not change the probability distribution, the reason of which is that the exponent of new datum N' satisfies

$$\log_{10} N' = \log_{10}(C \times N) = \log_{10} C + \log_{10} N. \qquad (5)$$

This property means that the law does not depend on any particular choice of units, namely scale-invariance[31], which was discovered by Pinkham in 1961. In mathematics, this law is the only digital law that is scale-invariant.

Thirdly, any power of the data set does not change the distribution either, i.e., the law is power-invariant. Because of $\log_{10} N' = \log_{10} N^\alpha = \alpha \log_{10} N$ for $\alpha \neq 0$, the logarithmic distribution remains unchanged according to the first property. In our research, the full widths of hadrons fit the logarithmic distribution, so do the lifetimes of hadrons.

Fourthly, the Benford's law is base-invariant[28,29,30] too, which means that it is independent of the base d you use. In the binary system (d=2), octal system (d=8), or other base system, the data, as well as in the decimal system (d=10), all fit the general first digit law,

$$P(k) = \log_d(1 + \frac{1}{k}), \ k = 1, 2, ..., d - 1. \qquad (6)$$

Hill proved strictly that "scale-invariance implies base-invariance"[28] and "base-invariance implies Benford's law"[29] mathematically in the framework of probability
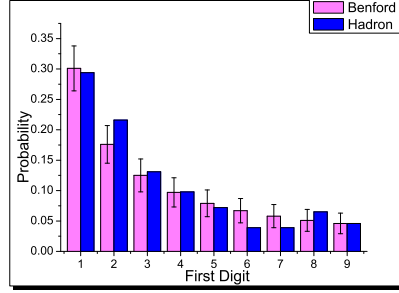
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Fig. 2.   Comparison of Benford's law and the first digit distribution of the full widths of hadrons in Case 1.
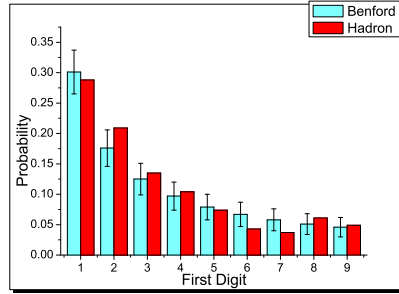

Fig. 3.   Comparison of Benford's law and the first digit distribution of the full widths of hadrons in Case 2.
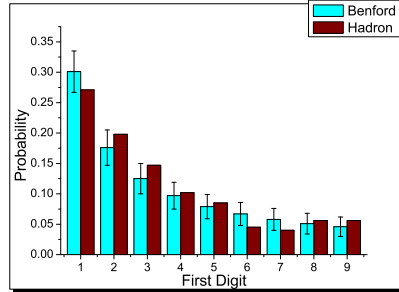

Fig. 4.   Comparison of Benford's law and the first digit distribution of the full widths of hadrons in Case 3.

theory.

Fifthly, in 2001, Pietronero et al.[32] provided a new insight, suggesting that a process or an object $N(t)$ with its time evolution governed by multiplicative fluctuations generates Benford's law naturally and they used stockmarket as a convictive example. Further, they demonstrated it with the computer simulation and got rather

good result. The main idea is that $N(t + \delta t) = r(t) \times N(t)$, where $r(t)$ is a random variable. After treating $\log r(t)$ as a new random variable, it is a Brownian process $\log N(t + \delta t) = \log r(t) + \log N(t)$ in the logarithmic space. Utilizing the central limit theorem in a large sample, $\log N(t)$ becomes uniformly distributed. Thus,

$$P(k) = \frac{\int_k^{k+1} \mathrm{d} \log N(t)}{\int_1^{10} \mathrm{d} \log N(t)} = \log_{10}(1 + \frac{1}{k}), \tag{7}$$

which is exactly the formula of Benford's law given in Eq. (1). This approach is well recommended in Refs. 5, 33.

In summary, we applied Benford's law to the well defined science domain of particle physics for the first time. The distribution of the first digits of the full widths of hadrons, including mesons and baryons respectively, all fit the logarithmic law remarkably well. Moreover, we discussed several general properties of the law, and reached the conclusion that our results apply to the lifetimes of hadrons as well. It is still a challenge to find the basic reason for the common distribution pattern among various sorts of natural behaviors. Our results suggest the necessity to look into some hitherto unnoticed features of basic physical phenomena. The first digit law can serve as a tool to test the reasonableness of any theory or model that is supposed to be the underlaying theory of the nature.

Acknowledgments

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